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IMPROVED ESTIMATORS OF K AND B IN FINITE POPULATIONS

SUKHMINDER SINGH and SARJINDER SINGH
Punjab Agricultural University, Ludhiana 141004

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SUMMARY

This paper proposes a class of estimators of K and B wider as well as more efficient than those considered by Srivastava, Jhajj and Sharma [5] using auxiliary information on a character x. An empirical investigation is given at the end.

Keywords : Auxiliary information; Ratio type estimators; Regression coefficient; Simple random sampling; Finite population; Asymptotic mean square error.

Introduction and Notations

Ratio type estimators have been very widely used for estimating the population mean $\bar{Y}$ of the study variable y using information on an auxiliary variable x. Among others, Srivastava [4] has defined a large class of estimators $\bar{Y}$. The optimum values of the parameters used were dependent upon $K = \rho\ C_y/C_x$ only. All the estimators belonging to this class of estimators, thus, involved K which is a function of population parameters. When the value of K is unknown, then it is required to be estimated from the given sample. Reddy [3] has also shown the stability of K than the linear regression coefficient $B = S_{xy}/S_x^2$ in repeated surveys. Thus, the estimation of K and B have independent importance in sample surveys.

Let $V = (1, 2, \ldots, N)$ be a finite population of N units and y be the study variable defined on V taking the value y_i for the ith unit of

V $(1 \leqslant i \leqslant N)$. Let x be an auxiliary variate correlated with y, taking the value x_i on the units $(1 \leqslant i \leqslant N)$, information on which is available in advance.

Let us assume that we draw a S.R.S. of size n from V. For simplicity let us assume that N is large as compared to n so that the finite population terms are ignored. Then write

$$\varepsilon_0 = \frac{\bar{y}}{\bar{Y}} - 1,\ \varepsilon = \frac{\bar{x}}{\bar{X}} - 1,\ \delta = \frac{s_x^2}{S_x^2} - 1,\ \eta = \frac{s_{xy}}{S_{xy}} - 1.$$

We have $E(\varepsilon_0) = E(\varepsilon) = E(\delta) = E(\eta) = 0$.

$$E(\varepsilon_0^2) = n^{-1} C_y^2,\ E(\varepsilon^2) = n^{-1} C_x^2,\ E(\varepsilon_0\varepsilon) = n^{-1} \rho C_y C_x,\ E(\varepsilon\delta) = n^{-1} C_x \lambda_{03},$$

$$E(\varepsilon_0\delta) = n^{-1} C_y \lambda_{12},\ E(\varepsilon_0\eta) = n^{-1} C_y \frac{\lambda_{21}}{\rho},\ E(\varepsilon\eta) = n^{-1} C_x \frac{\lambda_{12}}{\rho}$$

and upto terms of order n^{-1}

$$E(\delta^2) = n^{-1}(\lambda_{04} - 1),\ E(\eta^2) = n^{-1}\left(\frac{\lambda_{22}}{\rho^2} - 1\right)$$

$$E(\delta\eta) = n^{-1}\left(\frac{\lambda_{13}}{\rho} - 1\right)$$

Srivastava, Jhajj and Sharma [5] considered the following three estimators of K as

$$k_1 = \frac{\bar{x}}{\bar{y}} \frac{s_{xy}}{s_x^2},\ k_2 = \frac{\bar{X}}{\bar{Y}} \frac{s_{xy}}{s_x^2} \text{ and } k_3 = \frac{\bar{X}}{\bar{y}} \frac{s_{xy}}{S_x^2}$$

and MSEs of these estimators are given by

$$\text{MSE}(k_2) = \frac{K^2}{n}\Big[C_y^2 + C_x^2 - 2\rho C_y C_x - 2\lambda_{03} C_x + 2\lambda_{12} C_y + \lambda_{04} + \frac{\lambda_{22}}{\rho^2} + \frac{2\lambda_{12}}{\rho} C_x - \frac{2\lambda_{21}}{\rho} C_y - \frac{2\lambda_{13}}{\rho}\Big]$$

$$\text{MSE}(k_1) = \frac{K^2}{n}\left[C_y^2 + 2\lambda_{12} C_y + \lambda_{04} + \frac{\lambda_{22}}{\rho^2} - \frac{2\lambda_{21}}{\rho} C_y - \frac{2\lambda_{13}}{\rho}\right]$$

and $\text{MSE}(k_3) = \frac{K^2}{n}\left[C_y^2 - 1 + \frac{\lambda_{22}}{\rho^2} - 2\frac{\lambda_{21}}{\rho} C_y\right]$

Also they consider the two estimators of the population regression coefficient

$$B = \frac{S_{xy}}{S_x^2}, \quad \text{given by} \quad b_1 = \frac{s_{xy}}{s_x^2} \quad \text{and} \quad b_2 = \frac{s_{xy}}{S_x^2}$$

and MSEs of these estimators are given by

$$\text{MSE}(b_1) = \frac{B^2}{n}\left(\lambda_{04} - \frac{2\lambda_{13}}{\rho} + \frac{\lambda_{22}}{\rho^2}\right)$$

and $\text{MSE}(b_2) = \frac{B^2}{n}\left(\frac{\lambda_{22}}{\rho^2} - 1\right)$

Improved Estimators of $K = \rho\frac{C_y}{C_x}$

The three estimators of K are

$$k_{11} = -\frac{\bar{x}}{\bar{y}}\ \frac{s_{xy}}{s_x^2}\ h\left(\frac{\bar{x}}{\bar{X}}\right), \qquad k_{22} = \frac{\bar{X}}{\bar{y}}\ \frac{s_{xy}}{s_x^2}\ h\left(\frac{\bar{x}}{\bar{X}}\right)$$

and $k_{33} = \frac{\bar{X}}{\bar{y}} \cdot \frac{s_{xy}}{S_x^2}\ h\left(\frac{\bar{x}}{\bar{X}}\right)$.

where $h(.)$ is some parametric function such that $h(1) = 1$ and satisfy some regularity conditions.

Now k_{11} can be written as

$$k_{11} = \frac{\bar{X}}{\bar{Y}}\ \frac{S_{xy}}{S_x^2}\ [1 - \varepsilon_0 - \delta + \eta + \varepsilon + \varepsilon_0^2 + \varepsilon_0\delta - \varepsilon_0\eta - \delta\eta - \varepsilon_0\varepsilon - \varepsilon\delta + \varepsilon\eta + \delta^2 + \ldots]\ [1 + \varepsilon\, h_1(1)]$$

$$= K\,[1 - \varepsilon_0 - \delta + \eta + \varepsilon + \varepsilon_0^2 + \varepsilon_0\delta - \varepsilon_0\eta - \delta\eta - \varepsilon_0\varepsilon - \varepsilon\delta + \varepsilon\eta + \delta^2 + \varepsilon h_1(1) - \varepsilon\delta h_1(1) + \varepsilon\eta\, h_1(1) + \ldots]$$

Taking expectation and using the results from previous section, we get

$$E(k_{11}) = K + 0(n^{-1})$$

and MSEs upto terms of order n^{-1} are given by

$$M(k_{11}) = E[k_{11} - K]^2$$

$$= n^{-1} K^2 \Big[C_y^2 + C_x^2 - 2\rho C_y C_x - 2\lambda_{03} C_x + 2\lambda_{12} C_y + \lambda_{04} + \frac{\lambda_{22}}{\rho^2}$$

$$+ \frac{2\lambda_{12}}{\rho} C_x - \frac{2\lambda_{21}}{\rho} C_y - \frac{2\lambda_{13}}{\rho} - \left\{ C_x + \frac{\lambda_{12}}{\rho} - \rho Cy - \lambda_{03} \right\}^2 \Big]$$

$$= \text{MSE}(k_1) - \frac{K^2}{n} \left[C_x + \frac{\lambda_{12}}{\rho} - \rho C_y - \lambda_{03} \right]^2,$$

$$\text{where} \quad h_1(1) = \frac{-\left[C_x + \frac{\lambda_{12}}{\rho} - \rho C_y - \lambda_{03} \right]}{C_x}$$

Similarly it is easily found that the bias of k_{22} and k_{33} are of order n^{-1} and their mean squared errors upto order n^{-1} are given by

$$\text{MSE}(k_{22}) = \text{MSE}(k_2) - \frac{K^2}{n} \left(\frac{\lambda_{12}}{\rho} - \rho C_y - \lambda_{03} \right)^2,$$

$$\text{where} \quad h_1(1) = -\frac{\left(\frac{\lambda_{12}}{\rho} - \rho C_y - \lambda_{03} \right)}{C_x}$$

$$\text{and} \quad \text{MSE}(k_{33}) = \text{MSE}(k_3) - \frac{K^2}{n} \left(\frac{\lambda_{12}}{\rho} - \rho C_y \right)^2$$

$$\text{where} \quad h_1(1) = -\frac{\left(\frac{\lambda_{12}}{\rho} - \rho C_y \right)}{C_x}.$$

Improved Estimators of $B = \frac{S_{xy}}{S_x^2}$

The improved estimators of B are

$$b_{11} = \frac{s_{xy}}{s_x^2} h\left(\frac{\bar{x}}{\bar{X}}\right), \qquad b_{22} = \frac{s_{xy}}{S_x^2} h\left(\frac{\bar{x}}{\bar{X}}\right),$$

and proceeding as in previous section, obtain the MSEs of b_{11} and b_{22} given by

$$\text{MSE}(b_{11}) = \text{MSE}(b_1) - \frac{B^2}{n}\left(\frac{\lambda_{12}}{\rho} - \lambda_{03}\right)^2$$

where $$h_1(1) = -\frac{\left(\frac{\lambda_{12}}{\rho} - \lambda_{03}\right)}{C_x}$$

and $$\text{MSE}(b_{22}) = \text{MSE}(b_2) - \frac{B^2\lambda_{12}^2}{n\rho^2}$$

where, $$h_1(1) = -\frac{\lambda_{12}}{\rho C_x}$$

The efficiency comparisons of proposed estimators of K and B with the existing estimators are obvious from their MSEs expressions.

Numerical Illustration

For the purpose of numerical illustration of two populations described in Table 1 are taken in the study. In Table 2, the MSEs (upto terms of order n^{-1}) of the estimators k_i and b_i are compared, respectively, with the class of estimators k_{ii} and b_{ii} for $i = 1, 2, 3$.

TABLE 1 — DESCRIPTION OF POPULATIONS

Sr. No.	*Source*	*y*	*x*	ρ	C_y	C_x
1.	Horwitz and Thompson [1]	No. of household	Eye estimate of *y*	0.8662	0.4264	0.3889
2.	Murthy [2] p. 399 Vill : 1-10	Area under wheat (1964)	Area under wheat (1963)	0.9773	0.6187	0.5664

TABLE 2— MSEs OF k_i, k_{ii}, b_i and b_{ii} FOR $i = 1, 2, 3$ UPTO TERMS OF ORDER n^{-1}

Population No.	$1/n \times MSE$ of									
	k_1	k_{11}	k_2	k_{22}	k_3	k_{33}	b_1	b_{11}	b_2	b_{22}
1	0.3026	0.1767	0.5194	0.0049	1.3053	1.2325	0.6058	0.5262	2.0098	1.7901
2	0.0411	0.0409	0.3530	0.0020	1.4054	1.2595	0.0921	0.0904	1.9256	1.0392

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